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CHEATING PREVENTION ALGORITHMS IN EXAMINATIONS

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Abstract

In this paper, I will introduce methods to make test questions difficult for examinees to cheat in their examinations.

1. Introduction

On the National College Entrance Examination in Korea in 2005, a new type of cheating was detected, where some examinees received 'correct answers' via cellular phone text messages and wrote them on their test papers during the exam. This incident gave our society a great shock. As verifying the official announcement of the criminal investigation agency carefully, we may get the picture that a few students had already known this type of cheating in exams before.

As long as exams not only evaluate effective education, but also classify examinees based on their scores, the examinees are bound to cheat to raise their exam scores. One of matters to consider for examiners, thus, is 'how they may stop cheating?' when making exam questions.

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While examining Korean history, King Gwangjong of Goryeo Dynasty created the national civil service examinations to select and open the offices to the talented. The selected talent held powerful families in check so that the king could consolidate his sovereignty. There is no record of any cheating in the national civil service examinations from Goryeo Dynasty, but the following acts of cheating existed at the national civil service examination hall of Chosun Dynasty.

First, sons of the nobility seized an opportunity to bring several servants to the civil service examination place under loose supervision of proctors. The servants sneaked into the place with books, copied other examinees' test papers or changed test papers while getting in touch with the outside.

Second, official examiners (marking officers) happened to grade only the first half of submitted test papers, if there were too many examinees. Thus, examinees fought each other to submit their test papers before anyone else or bribed marking officers to insert their belated test papers on top of others quietly.

Third, sons from the influenced families bribed official examiners so that they could sneak away from the examination place, complete their test papers at home or schoolhouses and come back to the place to submit the test papers.

Fourth, some examinees brought five or six friends of theirs, who completed test papers each. Then, they selected and submitted the best test paper that is known as 'chasul,' an act of submitting another person's test paper as if it were his.

Fifth, some examinees bribed official examiners so that they could inform competent scholars waiting outside of the examination place, let them prepare substitute test papers and have the official examiners bring the test papers back to the examinees. This is known as 'daesul,' or an act of taking someone else's examination in place of him.

Sixth, some examinees marked a point, which had previously been promised between him and an official examiner, so that the official examiner would recognize his test paper and pass him.

Seventh, some examinees bribed a 'bongmigwan', an officer who verified the examinee's name, date of birth, and address and sealed his test paper, or a deputy official to seal their names instead of other successful candidates' names. This wicked method was known as 'jeolgwa,' or an act of stealing another person's test papers.

Eighth, examinees happened to act violently in the country, where they took examination places of 'hyangshi,' a regional preliminary examination before the national civil service examination, by assault and beat official examiners.

As mentioned above, the reason for frequent acts of cheating at the national civil service examination place was that passing the examination was the only way to raise their social status. Since students in Korea are exposed to intensive competition from childhood, they are unusually used to competing with others in every aspect. Some examinees in Korea tend to take all possible steps to raise their grades because they are compelled to receive higher grades than others at the time of exams. As long as our society remains as a society of limitless competition and methods to give questions in exams do not change significantly, cheating would not decrease in the future.

The aim of this study is to introduce methods to make test questions difficult for examinees to cheat in their examinations and humbly ask the readers to evaluate the effectiveness of the methods. Since I did not major in mathematical education, there is a chance that this paper may not follow the complete form of papers in education. Yet, I wish the readers would understand my sincere intention to decrease cheating in examinations through the methods introduced in this paper. If we could sufficiently prevent cheating in examinations, students would realize that accumulating their knowledge would benefit their future rather than cheat in examinations. Thus, such prevention of cheating in examinations will be extremely meaningful in education.

2. Cheating Prevention Algorithms

In this paper, I will introduce two algorithms that I have implemented and applied to those first year students taking mathematics class at the College of Science and Technology, Hongik University for four years from the spring 2004 to the fall 2007. The fundamental principles of the algorithms are as follows: As students begin their exams, they calculate their given m value in accordance with the algorithms and solve the questions using the m value.

In this paper, we will attempt to find algorithms with the following characteristics:

(1) Each student generates a different m value from one another.

(2) The algorithm that each student should follow is simple.

(3) Grading process is simple.

When presenting questions by using the above algorithm, the examiner must take the following precaution. If a student uses the letter m as it is in solving questions, other students may see it and do the same in their questions. The proctor should advise each student to use the number (m value) only rather than the letter m in their calculation.

2.1. First algorithm

(A) Assume that a is a natural number greater than or equal to 10. (We keep changing the value of a as we make questions.)

(B) Students multiply the first number of the last double-digit in their student identification number by a, and add the result to the number of last digit in their student identification number.

(C) Assume that m is a number in which a natural number c is added to the result in (B). (Here, the reason to add a natural number c is to prevent the value of m from becoming 0 or too small.)

When examining Theorem 1 and Problem 1, you may see that the first algorithm is very effective. Especially, using this algorithm results in 100 different m values. Since less than or equal to 100 students usually take an exam in a classroom, almost every student would take the exam with a different m value from one another.

Theorem 1. *Assume that the last two digits of two student identification numbers are different. Then, m values of the students generated by the first algorithm are different from each other.*

Proof. Assume that s_p and t_p are the first and last numbers of the last two digits in a student p's student identification number, respectively. Let s_q and t_q be the first and last numbers of the last two digits in another student q's student identification number, respectively. Then, the student p and the student q's m values are expressed as $m_p = as_p + t_p + c$ and $m_q = as_q + t_q + c$, respectively. We will use the reduction ad absurdum to prove this theorem. If $m_p = m_q$, then

$$0 = m_p - m_q = a(s_p - s_q) + (t_p - t_q) \text{ or } a(s_p - s_q) = t_q - t_p.$$

On the other hand, the second equation above yields $s_p = s_q$ and $t_p = t_q$ since $a \geq 10$, $-9 \leq s_p - s_q \leq 9$, and $-9 \leq t_q - t_p \leq 9$. In other words, the last two digits in the two students' identification numbers are the same. It is contrary to the assumption we have made in this theorem. We must negate the assumption $m_p = m_q$ that we have stated in accordance with the reductio ad absurdum; namely, $m_p \neq m_q$. □

We will apply the first algorithm in the following question with $a = 11$, $c = 3$.

Problem 2.1. Let m be the value in which you multiply the first number of the last double-digit in your student identification number by 11, add the result of the multiplication to the last digit in your student identification number, and add 3 to the result of the addition. Now calculate $\int_2^\infty \frac{1}{x(\ln x)^m} dx$.

As can be seen from Theorem 2.1, many students' m values are different from one another, Problem 2.1 becomes a slightly different question for each student. Since the answer to Problem 2.1

is $\frac{1}{m-1}\frac{\ln 2}{(\ln 2)^m}$, each student will yield a different answer from one another. Thus, it is not easy for a student to guess his/her answer by looking at another student's answer next to him/her.

2.2. Second algorithm

The second algorithm is based on the following theorem of the number theory.

Theorem 2. *If* $S(a, b)$ *is a set of any integers in the form of* $ma + nb$ *for given natural numbers a and b,* $S(a, b)$ *is a set of multiples of* (a, b). *Here, m and n are integers*, *whereas* (a, b) *is the greatest common divisor of a and b*.

If a and b are relatively prime in the above theorem, a linear combination of a and b, $S(a, b)$, becomes a set of multiples of 1. Then, $S(a, b)$ is $\mathbb{Z}$, the set of all integers.

The second algorithm may be explained as follows:

(A) Choose two natural numbers a and b that are relatively prime to each other. It is more convenient as a and b are smaller. (You should change the values of a and b for each exam.)

(B) Students multiply the first number and last number of the last double-digit in their student identification number by a and b, respectively, and add the two results.

(C) Assume that m is a number in which a natural number c is added to the result in (B). (Here, the values of m range from c to $c + 9$, which are integers.)

Using this algorithm results in 10 different m values. The advantage of the second algorithm yields simple values of m, whereas the disadvantage of the algorithm allows several students in a classroom to have the same value of m.

3. Classroom Case Studies

In classrooms, we have made mid-term and final examinations for College Mathematics course by using the second algorithm for eight semesters from the spring 2004 to the fall 2007 at the College of Science and Technology, Hongik University for four years from the spring 2004 to the fall 2007. The following is the analysis of the result.

3.1. Cheating in the examinations

Upon the class evaluation and individual interviews with students, cheating in the examinations almost disappeared since we began giving out exams by using the second algorithm in 2004. Students started to believe in the evaluation procedure of exams more so that complaint on their grades decreased significantly. Before applying this algorithm to the exams, some students contested their grades while leaving a message stating 'the professor could not deal with cheating in the exams adequately' in the course evaluation. Since 2004, we have hardly found any complaint on cheating from students.

3.2. Simple nature of the algorithm

As we held exams by using this algorithm, most students could calculate the value of m correctly upon simple instructions except two or three per 35 students on the average. Even if some students did not calculate the value of m correctly, we graded their exams in accordance with (wrong) values of m and such miscalculation did not change their grades significantly. Those students, who miscalculated the value of m correctly all scored 0. (The reason to grade their exams with wrong values of m is that some excellent students might have been greatly disadvantaged due to possible miscalculation of m.)

If students receive instructions on how to calculate the values of m for two or three times in class, hardly any students would not be able to find the values of m in the real exams.

3.3. Easy grading

Some may think grading exams by using the algorithm is complicated, since answers change along with the values of m. As we graded students' exams by using the above algorithm from 2004 to 2007 at Hongik University, it was not as difficult as some expected.

3.4. Precautions on the application of the algorithm

If the values of m change the method to solve questions completely, it is difficult to evaluate such questions objectively. It is recommended to avoid giving out such questions in exams; however, we have verified that only a few of these types of questions exist in real life.

The following is a bad example of such question.

Problem 3.1. Prove if the following infinite series $\sum_{n=1}^{\infty} \frac{1}{n^{m-5}}$ converges.

In order to solve the above question, students should use integral calculus, students with $m = 6$ and $m \neq 6$ have to calculate completely different integrals. Since this type of question makes it difficult to evaluate students objectively, it is desirable to avoid it.

4. Conclusion

Examinees are compelled to raise their grades even through cheating as long as the society does not stop ranking them based on their grades. One of the facts that an examiner must consider profoundly is 'how to prevent students from cheating in examinations?'

In this paper, I introduced 'cheating prevention algorithms' that we have proven the effect of the algorithms on the first year students at the College of Science and Technology, Hongik University taking the College Mathematics course. We could verify the fact that cheating in exams almost disappeared upon the class evaluation and individual interviews with students, since we began to apply the 'cheating prevention algorithms' in 2004. Students, moreover, started to believe in the evaluation procedure of exam more so that complaint on their grades decreased significantly.

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